① a) $(-\infty, 3) \Leftrightarrow x < 3$

b) $[-1, 4) \Leftrightarrow -1 \leq x < 4$

c) $E^*(2,2) \Leftrightarrow 0 < |x-2| < 2$

② a) $50 = 100 \cdot e^{-0,000121t} \Leftrightarrow 0,5 = e^{-0,000121t} \Leftrightarrow$

$\ln 0,5 = -0,000121t \Rightarrow t = \frac{\ln 0,5}{-0,000121} \simeq 5728,5$ años.

b) $2 = 100 \cdot e^{-0,000121t} \Rightarrow t = \frac{\ln 0,02}{-0,000121} \simeq 32.330,77$ años

③ a) $\frac{5x-1}{x+1} - \frac{2x+3}{x} = \frac{21}{2} \Leftrightarrow 2x(5x-1) - 2(x+1)(2x+3) =$

$= 21(x+1)x \Leftrightarrow 10x^2 - 2x - 4x^2 - 10x - 6 = 21x^2 + 21x \Rightarrow$

$\Rightarrow 0 = 15x^2 + 33x + 6 \Leftrightarrow 5x^2 + 11x + 2 = 0$

$x = \frac{-11 \pm \sqrt{121 - 40}}{10} = \frac{-11 \pm 9}{10} = \begin{cases} x_1 = -\frac{1}{5} \\ x_2 = -2 \end{cases}$

puesto que ninguna de las dos soluciones anula a ningún denominador, las dos soluciones son válidas.

b) $5x - \sqrt{x+2} = 3x + 2 \Leftrightarrow 2x - 2 = \sqrt{x+2} \Rightarrow$

$\Rightarrow (2x-2)^2 = x+2 \Leftrightarrow 4x^2 - 8x + 4 = x + 2 \Rightarrow$

$\Rightarrow 4x^2 - 9x + 2 = 0 \Rightarrow x = \frac{9 \pm \sqrt{81 - 32}}{8} =$

$= \frac{9 \pm 7}{8} \Rightarrow \begin{cases} \cancel{x = \frac{16}{8}} & x = 2 \\ \cancel{x = \frac{2}{8}} & x = \frac{1}{4} \end{cases}$

Si $x=2 \Rightarrow 5\cdot 2 - \sqrt{2+2} = 3\cdot 2 + 2$ válida.

Si $x=\frac{1}{4} \Rightarrow 5\cdot\frac{1}{4} - \sqrt{\frac{1}{4}+2} \neq 3\cdot\frac{1}{4}+2$ no válida.

$-0{,}25 \neq 2{,}75$

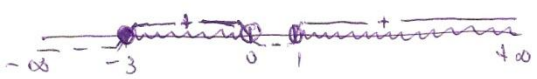
solución $\boxed{x=2}$

④ a) $3\cdot 9^x - 28\cdot 3^x + 9 = 0 \iff 3\cdot 3^{2x} - 28\cdot 3^x + 9 = 0$

$\underset{3^x=y}{\Longrightarrow} 3y^2 - 28y + 9 = 0 \Rightarrow y = \frac{28 \pm \sqrt{784-108}}{6}$

$y = \frac{28 \pm \sqrt{676}}{6} = \frac{28 \pm 26}{6} = \begin{cases} y_1 = 9 \Rightarrow 3^x = 9 \Rightarrow \boxed{x=2} \\ y_2 = \frac{1}{3} \Rightarrow 3^x = \frac{1}{3} \Rightarrow \boxed{x=-1} \end{cases}$

b) $\frac{x+3}{x^2-x} \geq 0 \Rightarrow \begin{cases} \text{E.a.n.: } x+3=0 \Rightarrow x=-3 \\ \text{E.a.d.: } x^2-x=0 \Rightarrow x(x-1)=0 \Rightarrow \begin{cases} x=0 \\ x=1 \end{cases} \end{cases}$



Sol.: $x \in [-3,0) \cup (1,+\infty)$

⑤ x = nº de pend. 1er proveedor.
y = nº de pend. 2º proveedor
z = nº de pend. 3º proveedor

$\Rightarrow \begin{cases} x+y+z = 170 \\ 6{,}1x + 6{,}2y + 6{,}3z = 1051 \\ y = 2x \end{cases}$

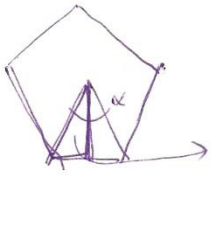
$\Rightarrow \begin{cases} x+y+z = 170 \\ 6{,}1x + 6{,}2y + 6{,}3z = 1051 \\ 2x - y = 0 \end{cases} \Rightarrow \begin{pmatrix} E_1 \\ E_2 - 6{,}3E_1 \\ E_3 \end{pmatrix} \Rightarrow \begin{cases} x+y+z = 170 \\ -0{,}2x - 0{,}1y = -20 \\ 2x - y = 0 \end{cases}$

$\Rightarrow \begin{pmatrix} E_1 \\ -10E_2 \\ E_3 \end{pmatrix} \Rightarrow \begin{cases} x+y+z = 170 \\ 2x + y = 200 \\ 2x - y = 0 \end{cases} \Rightarrow \begin{cases} E_1 \\ E_2 \\ E_3 + E_2 \end{cases} \Rightarrow \begin{cases} x+y+z = 170 \\ 2x+y = 200 \\ 4x = 200 \end{cases}$

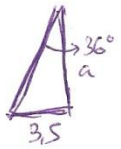
$$\Rightarrow \begin{cases} \longrightarrow & 50 + 100 + z = 170 \Rightarrow z = 20. \\ \Uparrow & \\ \Longrightarrow & 2\cdot 50 + y = 200 \Rightarrow y = 100 \\ \Uparrow & \\ x = \frac{200}{4} = 50 & \end{cases}$$

$$\boxed{(x, y, z) = (50, 100, 20).}$$

(6)



$\alpha = \frac{360°}{5} = 72°$ $\qquad \alpha/2 = 36°.$



$$tg\,36° = \frac{3,5}{a} \Rightarrow a = \frac{3,5}{tg\,36°}$$

$$a = 4,8173$$

(7) $tg\,\alpha = -2 \Rightarrow 1 + tg^2\alpha = \sec^2\alpha \Rightarrow 1 + (-2)^2 = \sec^2\alpha$

$$\Rightarrow \sec^2\alpha = 5 \Rightarrow \cos^2\alpha = \frac{1}{5} \Rightarrow \boxed{\cos\alpha = -\frac{1}{\sqrt{5}}}$$

$$\Downarrow \boxed{\sec\alpha = -\sqrt{5}}$$

$$\boxed{\operatorname{sen}\alpha = tg\,\alpha \cdot \cos\alpha = (-2)\cdot\left(-\frac{1}{\sqrt{5}}\right) = \frac{2}{\sqrt{5}}}$$

$$\boxed{\operatorname{cosec}\alpha = \frac{\sqrt{5}}{2}} \qquad \boxed{\operatorname{cotg}\,\alpha = -\frac{1}{2}}$$

(8) a) $2\cos x = \sec x \iff 2\cos x = \frac{1}{\cos x} \iff$

$$\iff 2\cos^2 x = 1 \Rightarrow \cos^2 x = \frac{1}{2} \Rightarrow \cos x = \pm\frac{1}{\sqrt{2}} = \pm\frac{\sqrt{2}}{2}$$

Si $\cos x = \frac{\sqrt{2}}{2} \Rightarrow \begin{cases} x_1 = \frac{\pi}{4} + 2k\pi \\ \vdots \\ x_2 = -\frac{\pi}{4} + 2k\pi \end{cases}$

Si $\cos x = -\frac{\sqrt{2}}{2} \Rightarrow \begin{cases} x_3 = \frac{3\pi}{4} + 2k\pi \\ x_4 = -\frac{3\pi}{4} + 2k\pi. \end{cases}$

b) $\cos x = \text{sen} 2x \iff \cos x = 2 \,\text{sen}\, x \cos x \iff$

$0 = 2 \,\text{sen}\, x \cos x - \cos x \iff 0 = (2 \,\text{sen}\, x - 1) \cdot \cos x \Rightarrow$

$\Rightarrow \begin{cases} \cos x = 0 \\ \text{ó} \\ 2 \,\text{sen}\, x - 1 = 0 \end{cases}$

$\cos x = 0 \Rightarrow \begin{cases} x_1 = \frac{\pi}{2} + 2k\pi \\ x_2 = -\frac{\pi}{2} + 2k\pi \end{cases}$

$2 \,\text{sen}\, x - 1 = 0 \Rightarrow \text{sen}\, x = \frac{1}{2} \Rightarrow \begin{cases} x_3 = \frac{\pi}{6} + 2k\pi \\ x_4 = \frac{5\pi}{6} + 2k\pi \end{cases}$